University of Florida
Mathematics Department
2008-09 CENTER FOR APPLIED MATH
COLLOQUIUM

by

Bertram Kostant *
Massachusetts Institute of Technology

on

Some Exotic Finite Subgroups of \(E_8\) and certain 8th-
Degree Cyclotomic Polynomials

Date: Tuesday, April 14 (2008)
Time: 4:00 - 5:30pm
Room: LIT 339
Refreshments: Before the lecture in the Atrium (LIT 339)

OPENING REMARKS
by
TBA

Abstract. (joint work with N. Wallach).

Let \(g\) be a complex simple Lie algebra and let \(G\) be the adjoint group of \(g\).
Let \(h\) be the Coxeter number of \(g\). Some time ago I conjectured that if \(q = 2h+1\) is a prime power, then the finite simple group \(L_2(q)\) embeds into \(G\). With the help of computers, in a number of cases, this has been shown to be true. The most sophisticated case is when \(G = E_8\). Here \(q = 61\). This embedding was first computer-established by Cohen–Griess and later without computer by Serre. Griess–Ryba also later (computer) proved that \(L_2(49)\) and \(L_2(41)\) embed into \(E_8\).

Write the three power primes 61, 49, 41 as \(q_k\) where \(k = 30, 24, 20\) so that \(q_k = 2k + 1\). In a 1959 paper I related, for any simple \(g\), the Coxeter element with the principal nilpotent element in \(g\). Tony Springer, in a 1974 paper, extending my result in the special case of \(E_8\), established a similar connection, between three nilpotent elements, \(e_k\) in \(g\), and three (regular) elements of the Weyl group \(\sigma_k\). The order of \(\sigma_k\) is \(k\). Using some beautiful properties of \(\sigma_k\) the main result in our presentation here at is the establishment of a clear-cut connection between Springer’s result, on the one hand, with the Griess-Ryba embedding \(L_2(q_k)\) in \(E_8\) on the other.

* Professor Bertram Kostant is a world authority in representation theory. His fundamental research spans several areas such as Lie groups and Lie algebras, homogeneous spaces, differential geometry, and mathematical physics. He is known for defining a quantization procedure now called pre-quantization, and for developing a complete theory of quantum Toda lattices in which his quantization program can be achieved. After receiving his PhD from Chicago in 1954, he was on the faculty at Berkeley before moving to MIT in 1962 where he has ever since. He has received numerous honors and recognitions in his long illustrious career. He is a Member of the National Academy of Sciences and the American Academy of Arts and Sciences. In 1997 he received an Honorary Doctorate from Purdue where he did his undergraduate
studies.